

§3-1. ELEMENTARY MATRIX OPERATIONS and ELEMENTARY MATRICES

Def'n Elementary Row Operations

- $A \in M_{m \times n}(\mathbb{F})$
- TYPE 1** → ① interchanging any two rows of A
- TYPE 2** → ② multiplying any row of A by a non-zero scalar
- TYPE 3** → ③ adding any scalar multiple of a row of A to another row

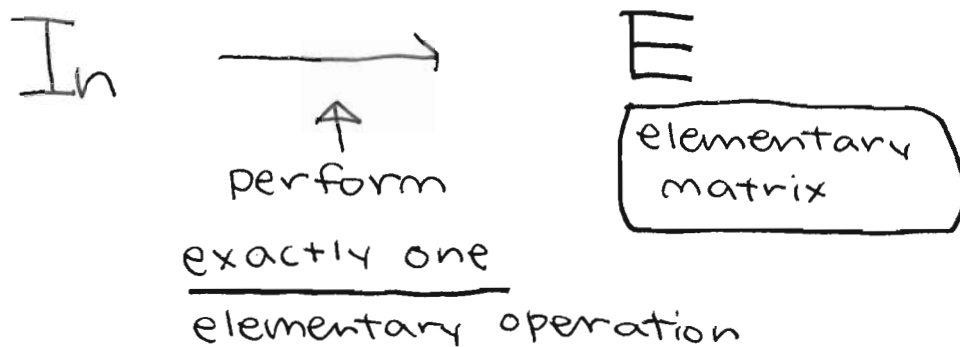
eg 1.

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & -1 & 3 \\ 4 & 0 & 1 & 2 \end{pmatrix}$$

TYPE 1
→ $2 \times ①$ $\begin{pmatrix} 2 & 1 & -1 & 3 \\ 1 & 2 & 3 & 4 \\ 4 & 0 & 1 & 2 \end{pmatrix}$

TYPE 2
→ $\begin{pmatrix} 4 & 2 & -2 & 6 \\ 1 & 2 & 3 & 4 \\ 4 & 0 & 1 & 2 \end{pmatrix}$

Elementary matrix



eg. 1

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

在矩陣 A 上要作某個基本列運算，只要用相對應之基本矩陣 E 乘以 A 即可！

eg. 1

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 4 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \\ 4 & 0 & 1 \end{pmatrix}$$

Thm 3.1

$$A \in M_{m \times n}(F)$$

$$\left. \begin{array}{l} A \xrightarrow{\text{elementary operation}} B \\ \text{same} \\ I_m \rightarrow E \end{array} \right\} \Rightarrow EA = B$$

Thm 3.2

Elementary matrices are invertible and the inverse of an elementary matrix is an elementary of the same type.

Pf.

TYPE 1: $E^{-1} = E$

TYPE 2: $E =$ multiplying any row of I_n by a non zero scalar α

$$E^{-1} = \begin{array}{ccc} & & \\ & & \\ \hline & & \frac{1}{\alpha} \\ & & \end{array}$$

TYPE 3: $E =$ ~~multiplying~~ adding any scalar multiple α of a row of I_n to another row

$$E^{-1} = \begin{array}{ccc} & & \\ & & \\ \hline & & \\ \hline & & \end{array}$$

eg.

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$